

**A STOCHASTIC STUDY OF INCARCERATION TIMES FOR NARCOTIC  
DISTRIBUTORS IN A CITY UNDER THE "THREE STRIKE" LAW**

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Specific questions that we are address include: Do a fixed set of incarceration times set trends in drug dealer populations? For a fixed set of incarceration times, how much does it cost to jail drug dealers 5 years? For 10 years? What do these costs buy? Are there less drug dealers as a result of these policies?

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# A Stochastic Study of Incarceration Times for Narcotic Distributers in a City Under “The Three Strike Law”

Jaime Barrera <sup>\*</sup>      Joaquin Rivera-Cruz <sup>†</sup>      Carlos W. Melgar <sup>‡</sup>

## Abstract

Narcotics ruins peoples' lives in the United States everyday. The United States government not only imposes stiff sentences on drug dealers but also spends billions of dollars a year developing programs to try to reduce the high intensity of drug distribution in cities across the United States. Unfortunately there is still a high number of narcotics being distributed within the United States. We use a stochastic model to study the trends of drug dealer populations set into motion as a result of fixing a set of incarceration times for drug dealers. We then project these trends and calculate costs associated with jailing drug dealers and the associated active drug dealer distributions.

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## Introduction:

### National Facts

The use and distribution of illicit drugs is one of the biggest and most expensive problems the United States government faces today. In 1995, the US government spent a total of \$ 29.7 billion dollars in the war against drug manufacturing and distribution (Drug Policy Foundation). From 1985 to

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1997, the total budget spent to fund programs against drug manufacturing and distribution increased by more than 50% (web.NIDA). The increase is due to many factors, one such being criminal activity.

Drug manufacturing and distribution is a problem that is getting worse due to the direct relationship between drugs and crime. Although this relationship is not completely understood, recent statistical studies have shown that this relationship can be summarized in three points: Drug users report greater involvement in crime and are more likely than nonusers to have criminal records; persons with criminal records are much more likely than those without criminal records to report being drug users; lastly, since drug dealers use violence as a method to protect, expand their drug market, reduce competition, and to intimidate the police officials. Crime rises as drug use and distribution rises.

The problem is acute in urban areas. In a sample of 415 homicides in New York City during 1989, 53% were drug related (U.S. Dept. of Justice). In a public study made during 1989, 58% of people believed that drugs were the main factors responsible for crime ( U.S. Dept. of Justice).

Even though drug use and distribution have been mentioned as one of the most important problems facing the nation today, little has been done to try to control this epidemic.

## California: Drugs and Incarceration Issue

The state of California is one of the areas with the highest rates of drug use and distribution. A 1991 FBI report showed that California has a proportion of drugs related arrest of 500+ per 100,000 population (U. S. Dept. of Justice). The prison population for the 1997-year was 163,695. About 65% of all the inmates in California are committed to prison from Southern California, with about 36% from Los Angeles County, 8% from San Diego County, and 15% from the San Francisco Bay area (CA Dept. of Corrections). The California State Department of Corrections (CDC) thinks that it will have 232,386 inmates in the year 2001. The proposed 1996-1997 CDC budget was about 3.8 billion dollars, which was a 11% (360 million dollar) increase from the previous year (CA Dept. of Corrections).

California enforces a "Three Strikes and You're Out" policy. The purpose of "Three Strikes" is to put repeating felons behind bars for a longtime. Specifically, "threepeaters" for a minimum of 25 years after their third arrest. If an individual commits a total of three offenses, the individual would

be put in jail for a minimum of twenty-five years. CDC's forecast assumes that the "Three Strikes and You're Out" law enacted in 1994 will continue to have a great impact in increasing the prison population. During the 1996-97 one of every eight offenders (9,628) sent to prison under the "Three Strikes" law was a third time offender (CA Dept. of Corrections). The CDC expect 13,100 "Three Strikes" admissions annually by 2000-2001, this implies a increase of millions of dollars to the budget.

## MODEL

To help simplify the analysis of the problem, we apply the "Three Strikes and You're Out" policy enforced in California in a simple stochastic model. In our system we consider only drugs dealers who move from society to jail back to society until their third arrest. Our purpose is to estimate the minimum incarceration time for first and second time offenders. We want the minimum time, because if we now this information we then can project these trends and calculate costs associated with jailing drug dealers and the associated active drug dealer distributions.

### Definitions:

**Dealer:** Any individual that distributes any type of narcotics.

**Drug Offence:** A misdemeanor or felony for distributing drugs resulting in incarceration.

**Fresh Dealer:** A drug dealer that has never been arrested.

**First Time Offender:** A drug dealer in jail that has been arrested only once.

**Once-Released Offender:** A drug dealer who has been incarcerated once and has been released. Once-released offenders continue to deal drugs.

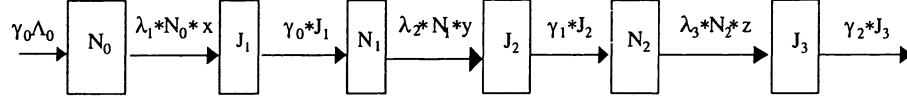
**Second Time Offender:** A drug dealer in jail that has been arrested exactly twice.

**Twice-Released Offender:** A drug dealer who has been incarcerated twice and has been released twice. Twice-released offenders continue to deal drugs.

**Third Time Offender:** A drug dealer in jail that has been arrested exactly thrice.

**Thrice-Released Offender:** A drug dealer who has been incarcerated three and has been released three times. Thrice-released offenders do not continue to deal drugs.

Model:



where  $x = e^{-\frac{\alpha_0}{\gamma_0}}$ ,  $y = e^{-\frac{\alpha_1}{\gamma_1}}$ , and  $z = e^{-\frac{\alpha_2}{\gamma_2}}$

Assumptions:

- After the individual is released from jail for the third time, the individual leaves the system completely, i.e. does not deal drugs anymore.
- After every arrest, the average time for a drug dealer to get arrested increases due to experience. Average time before a drug dealer gets arrested for the:

first time is between 0-1 year  
second time is between 1-2 years  
third time is between 2-5 years

- After a dealer is arrested and subsequently released from jail, he/she continues to distribute drugs.
- When a dealer is in jail, he/she cannot deal drugs. So we do not consider him/her a threat to society.
- The number of arrests a law official makes is not affected by the number of dealers in jail.
- Dealers can only be arrested for dealing drugs.
- Jails are big enough to fit all the arrested dealers. So we are not concerned with lack of jail space.
- The inflow of new dealers into our system depends only on the the average first incarceration time.
- The rates at which dealers are arrested are decreasing functions of the average incarceration times.

Parameters:

$N_0$  is the number of fresh dealers.  
 $N_1$  is the number of once-released offenders.  
 $N_2$  is the number of twice-released offenders.  
 $J_1$  is the number of first time offenders.

$J_2$  is the number of second time offenders.

$J_3$  is the number of twice time offenders.

$\Lambda = \gamma_0 \Lambda_0$  is the inflow of dealers, i.e. people starting careers in dealing drugs.

$\frac{1}{\lambda_1}$  is the average time before a fresh dealer gets arrested for dealing drugs.

$\frac{1}{\gamma_0}$  is the average time in jail for a first time offender.

$\frac{1}{\lambda_2}$  is the average time before a once-released offender gets arrested again for dealing drugs.

$\frac{1}{\gamma_1}$  is the average time in jail for a second time offender.

$\frac{1}{\lambda_3}$  is the average time before a twice-released offender gets arrested again for dealing drugs.

$\frac{1}{\gamma_2}$  is the average time in jail for a third time offender.

Equations:

$$N'_o = \gamma_0 \Lambda_0 - \lambda_1 N_o e^{-\frac{\alpha_0}{\gamma_0}} \quad (1)$$

$$J'_1 = \lambda_1 N_o e^{-\frac{\alpha_0}{\gamma_0}} - \gamma_o J_1 \quad (2)$$

$$N'_1 = \gamma_o J_1 - \lambda_2 N_1 e^{-\frac{\alpha_1}{\gamma_1}} \quad (3)$$

$$J'_2 = \lambda_2 N_1 e^{-\frac{\alpha_1}{\gamma_1}} - \gamma_1 J_2 \quad (4)$$

$$N'_2 = \gamma_1 J_2 - \lambda_3 N_2 e^{-\frac{\alpha_2}{\gamma_2}} \quad (5)$$

$$J'_3 = \lambda_3 N_2 e^{-\frac{\alpha_2}{\gamma_2}} - \gamma_2 J_3 \quad (6)$$

Estimating Parameters:

Estimating our parameters was one of the many problems we faced in developing our model. The problem we with the parameters in our model was that we could not get access to some actual data. For example the average time spend before a drug dealer get's arrested for the  $i^{\text{th}}$  time,  $\frac{1}{\lambda_i}$  for  $i = 1, 2, 3$ , is one of parameter that we had to estimate, since we could not find this information in any statistical reference.

We talked with a couple police officers from Los Angeles county to help us set  $\frac{1}{\lambda_i}$ . Both officers were in their respective narcotics divisions and for their safety and the safety of their families did not give their names. Each one independently suggested the parameter ranges. Specifically  $0 \leq \frac{1}{\lambda_1} \leq 1$  years,  $1 \leq \frac{1}{\lambda_2} \leq 2$  years, and  $2 \leq \frac{1}{\lambda_3} \leq 5$  years.

The next parameters to estimate was intial population for drug dealers in jail for their first, second, and third time. We assumed, for simplicity of our model, at time equal to zero the initial drug dealers in jail for the first, second, and third time were all equal to 100 dealers,  $J_0 = J_1 = J_3 = 100$ . Also it is assumed that at time equal to zero, the initial number of drug dealers that have been arrested for the first, second, and third time are all equal to 200 dealers,  $N_0 = N_1 = N_2 = 200$ .

The next parameter to estimate was the number of new drug dealers per unit of time,  $\gamma_0 \Lambda_0$ , at time equal to zero. To ensure that for plausible incarceration times we get a number of fresh dealers,  $\Lambda$ , that is realistic in comparison to the initial population sizes we set  $\Lambda_0$  equal to 650 dealers.

In modeling the number of new drug dealers coming into our system,  $\Lambda$ , we consider the fact that if a drug dealer is put in jail for a long time for the first offence, this would discourage new drug dealers from coming into the system. And vice-versa, if a drug dealer is put in jail for a short period of time, this would encourage more people to deal drugs and therefore come into the sytem. So we let  $\Lambda$  be a function satisfying the above mentioned conditions.

We fix the average time in jail for a third time drug offender,  $\frac{1}{\gamma_2}$ , to be equal to 25 years because of the "Three Strikes" law. The purpose of "Three Strikes" is to put repeating felons behind bars for a long time, i.e. for a minimum of 25 years after their third arrest.

We have only have  $\alpha_i$  left to estimate, since we plan to vary the average incarceration times,  $\frac{1}{\gamma_0}$ ,  $\frac{1}{\gamma_1}$ . For simplicity's sake we let  $\alpha_1 = \alpha_2 = \alpha_3$ . We noticed after varing  $\alpha_i$  from 0.1,  $1 \times 10^{-5}$ ,  $1 \times 10^{-6}$ , and  $1 \times 10^{-11}$  that the distributions were uninteresting for some cases ( $\alpha_i = 0.1$ ) and relatively the same for other choices of  $\alpha_i$ .

For  $\alpha_i = 0.1$  our jail populations tend to zero after a short period of time, roughly 5 years. See Figure 1.1 and 1.2.

**Note:** All simulations were run ten times and for thirty years.

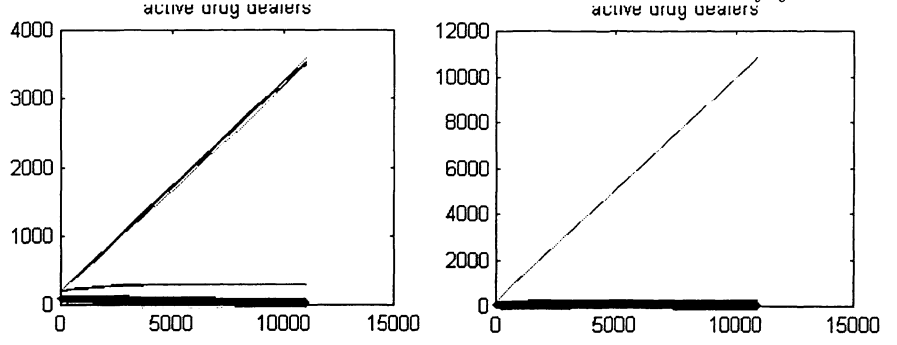


Figura 1.1

Figura 1.2

Figure 1.1  $(\frac{1}{\gamma_o}, \frac{1}{\gamma_1}) = (5,5)$  and Figure 1.2  $(\frac{1}{\gamma_o}, \frac{1}{\gamma_1}) = (0.5,10)$  almost all populations go to zero especially the jail populations for  $\alpha_i = 0.1$ . In both figures only a few dealer populations survive.

For  $\alpha_i = 1 \times 10^{-5}$  (Figure 1.3),  $1 \times 10^{-6}$  (Figure 1.4), and  $1 \times 10^{-11}$  (Figure 1.5) we get similar resulting distributions.

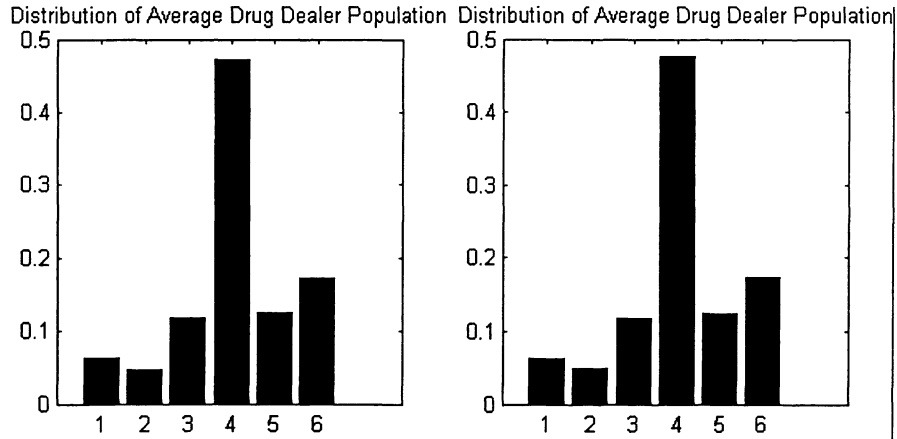


Figura 1.3

Figura 1.4



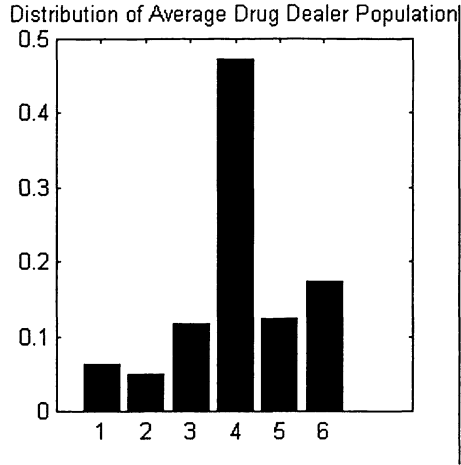


Figura 1.5

Figure 1.3 for  $(\frac{1}{\gamma_o}, \frac{1}{\gamma_1})=(0.5,15)$ , Figure 1.4 for  $(\frac{1}{\gamma_o}, \frac{1}{\gamma_1}) = (0.5, 15)$  and Figure 1.5 for  $(\frac{1}{\gamma_o}, \frac{1}{\gamma_1}) = (0.5, 20)$  show the resulting populations to be very similar. The numbers correspond as  $1 = N_0, 2 = J_1, 3 = N_1, 4 = J_2, 5 = N_2, 6 = J_3$ .

Since we are looking for interesting trends in the populations we disregard  $\alpha_i=0.1$  and arbitrarily choose  $\alpha_i = 1 \times 10^{-5}$  instead of  $1 \times 10^{-6}$  or  $1 \times 10^{-11}$ .

## Deterministic Analysis:

The deterministic approach gives us an explicit solution to the problem, but the results of a stochastic model adds the realistic property of chance to the dynamics.

$$\text{If } A = \begin{bmatrix} -\lambda_1 & 0 & 0 & 0 & 0 & 0 \\ \lambda_1 & -\lambda_1 e^{-\frac{\alpha_0}{\gamma_0}} & 0 & 0 & 0 & 0 \\ 0 & \lambda_1 e^{-\frac{\alpha_0}{\gamma_0}} & -\lambda_2 & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & -\lambda_2 e^{-\frac{\alpha_1}{\gamma_1}} & 0 & 0 \\ 0 & 0 & 0 & \lambda_2 e^{-\frac{\alpha_1}{\gamma_1}} & -\lambda_3 & 0 \\ 0 & 0 & 0 & 0 & \lambda_3 & -\lambda_3 e^{-\frac{\alpha_2}{\gamma_2}} \end{bmatrix} \text{ and}$$

$$B = \begin{bmatrix} \Lambda_0 \gamma_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \text{ then } \bar{x}'(t) = A\bar{x} + \bar{B} \text{ with } x = \begin{bmatrix} N_0 \\ J_1 \\ N_1 \\ J_2 \\ N_2 \\ J_3 \end{bmatrix} \text{ and the solution}$$

is  $\bar{x}(t) = (\bar{X}_0 - A^{-1}\bar{B})e^{-At} - A^{-1}\bar{B}$  where  $\bar{X}_0$  is an initial population distribution and where  $A^{-1}$  exists since  $A$  is an upper diagonal matrix with diagonal elements different from zero. Since  $A$  is upper diagonal matrix then the diagonal elements are the eigenvalues. Notice that all the eigenvalues are negative, so  $\lim_{t \rightarrow \infty} \bar{x}(t) = -A^{-1}\bar{B}$ .

We now compute  $A^{-1}\bar{B}$ .

$$A^{-1}\bar{B} = \begin{pmatrix} -\frac{1}{\lambda_1} & 0 & 0 & 0 & 0 & 0 \\ -\frac{1}{e^{-\frac{\alpha_0}{\gamma_0}} \lambda_1} & -\frac{1}{e^{-\frac{\alpha_0}{\gamma_0}} \lambda_1} & 0 & 0 & 0 & 0 \\ -\frac{1}{e^{-\frac{\alpha_1}{\gamma_1}} \lambda_2} & -\frac{1}{e^{-\frac{\alpha_1}{\gamma_1}} \lambda_2} & -\frac{1}{e^{-\frac{\alpha_1}{\gamma_1}} \lambda_2} & 0 & 0 & 0 \\ -\frac{1}{e^{-\frac{\alpha_2}{\gamma_2}} \lambda_3} & -\frac{1}{e^{-\frac{\alpha_2}{\gamma_2}} \lambda_3} & -\frac{1}{e^{-\frac{\alpha_2}{\gamma_2}} \lambda_3} & -\frac{1}{e^{-\frac{\alpha_2}{\gamma_2}} \lambda_3} & 0 & 0 \\ -\frac{1}{e^{-\frac{\alpha_2}{\gamma_2}} \lambda_3} & -\frac{1}{e^{-\frac{\alpha_2}{\gamma_2}} \lambda_3} & -\frac{1}{e^{-\frac{\alpha_2}{\gamma_2}} \lambda_3} & -\frac{1}{e^{-\frac{\alpha_2}{\gamma_2}} \lambda_3} & -\frac{1}{e^{-\frac{\alpha_2}{\gamma_2}} \lambda_3} & 0 \\ -\frac{1}{e^{-\frac{\alpha_2}{\gamma_2}} \lambda_3} & -\frac{1}{e^{-\frac{\alpha_2}{\gamma_2}} \lambda_3} & -\frac{1}{e^{-\frac{\alpha_2}{\gamma_2}} \lambda_3} & -\frac{1}{e^{-\frac{\alpha_2}{\gamma_2}} \lambda_3} & -\frac{1}{e^{-\frac{\alpha_2}{\gamma_2}} \lambda_3} & -\frac{1}{e^{-\frac{\alpha_2}{\gamma_2}} \lambda_3} \end{pmatrix} \begin{pmatrix} \Lambda_0 \gamma_0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

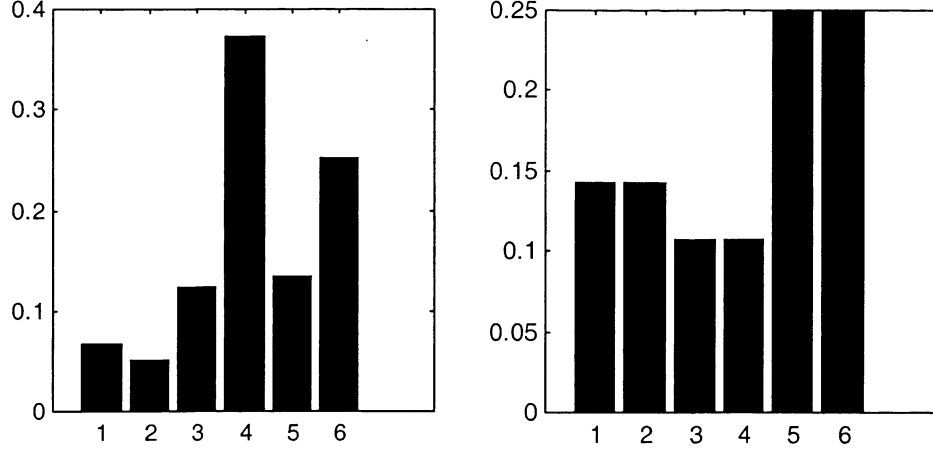
$$= \begin{pmatrix} -\frac{\Lambda_0 \gamma_0}{\lambda_1} \\ -\frac{\Lambda_0 \gamma_0 e^{-\frac{\alpha_0}{\gamma_0}}}{\lambda_1} \\ -\frac{\Lambda_0 \gamma_0}{\lambda_2} \\ -\frac{\Lambda_0 \gamma_0 e^{-\frac{\alpha_1}{\gamma_1}}}{\lambda_2} \\ -\frac{\Lambda_0 \gamma_0}{\lambda_3} \\ -\frac{\Lambda_0 \gamma_0 e^{-\frac{\alpha_2}{\gamma_2}}}{\lambda_3} \end{pmatrix}$$

For the above choices of parameters and  $(\frac{1}{\gamma_0}, \frac{1}{\gamma_1}) = (0.5, 10)$

$$\lim_{t \rightarrow \infty} \bar{x}(t) = -A^{-1}\bar{B} = \begin{pmatrix} 2600 \\ 2600 \\ 1950 \\ 1950 \\ 4550 \\ 4550 \end{pmatrix}.$$

The next figures show (left) the average distribution for drug dealers and (right) the limiting deterministic distribution for drug dealers.

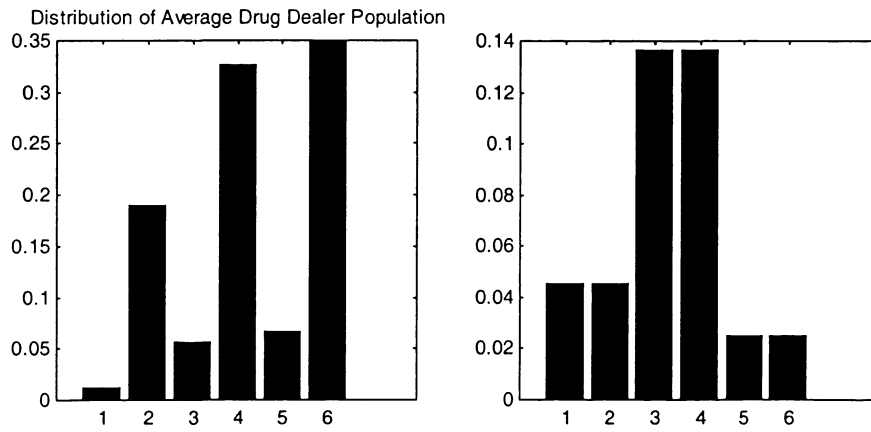
Distribution of Average Drug Dealer Population



And now for the above choices of parameters and  $(\frac{1}{\gamma_o}, \frac{1}{\gamma_i}) = (5, 10)$

$$\lim_{t \rightarrow \infty} \bar{x}(t) = -A^{-1}\bar{B} = \begin{pmatrix} 65 \\ 65 \\ 195 \\ 195 \\ 455 \\ 455 \end{pmatrix}.$$

The next figures show (left) the average distribution for drug dealers and (right) the limiting deterministic distribution for drug dealers.



As you can see the distributions from the deterministic results have little variation, in fact they come in pairs  $(N_0, J_1)$ ,  $(N_1, J_2)$  and  $(N_2, J_3)$ . And although our simulations do not seem to be limiting to the deterministic distributions, we cannot rule out the possibility that they do not. The two may not match because the deterministic distribution may limit after a time that may be unrealistic.

## Methodology

We are interested in finding out what trends result when public policy dictates incarceration times in a city that enforces the “three strikes and you’re out” policy. As we would like to predict future criminal trends and their resulting jail costs.

Our first step in this process is to fix the set of incarceration times,  $(\frac{1}{\gamma_0}, \frac{1}{\gamma_1})$ . Next we wanted to simulate the solutions to Equations 1-6 with a stochastic simulation model.

So we must now examine the possible events.

Event 1: A person moves into the fresh dealer population,  $N_0 \Rightarrow N_0+1$  at the rate  $\Lambda$ .

Event 2: A person is removed from the fresh dealer population and added to the first time offender,  $(N_0, J_1) \Rightarrow (N_0-1, J_1+1)$ , at the rate  $\lambda_1 N_0 e^{-\frac{\alpha_0}{\gamma_0}}$ .

Event 3: A person is removed from the first time offender population and added to the once-released population,  $(J_1, N_1) \Rightarrow (J_1-1, N_1+1)$ , at the rate  $\gamma_0 J_1$ .

Event 4: A person is removed from the once-released offender population and added to the second time offender,  $(N_1, J_2) \Rightarrow (N_1-1, J_2+1)$ , at the rate  $\lambda_2 N_1 e^{-\frac{\alpha_1}{\gamma_1}}$ .

Event 5: A person is removed from the second time offender population and added to the twice-released population,  $(J_2, N_2) \Rightarrow (J_2-1, N_2+1)$ , at the rate  $\gamma_1 J_2$ .

Event 6: A person is removed from the twice-released offender population and added to the third time offender,  $(N_2, J_3) \Rightarrow (N_2-1, J_3+1)$ , at the rate  $\lambda_3 N_2 e^{-\frac{\alpha_2}{\gamma_2}}$ .

Event 7: A person is removed from the third time offender population,  $J_3 \Rightarrow J_3-1$ , at a rate  $\gamma_2 J_3$ .

Now we suppose that for each event the time between occurrences is exponentially distributed. We discretize time into days, a small increment

of time when one considers that the dynamics at which we will be looking occur over tens of years. Also we assumed that the events are independent of each other. This is plausible, i.e. a prisoner being released from jail does not, as a single event in a day, affect whether or not Johnny Hood becomes a drug dealer. So at the end of each day each one of these events has had the probability  $1 - e^{-P(\text{rate of event } i)}$  of occurring. This means that any combination of any number of events 1-7 to all seven events can happen in a day. When the computer uses a random number generator to decide what happens, this algorithm decides the dynamics of the system stochastically.

Using the above algorithm we ran the ten simulations for thirty years for each of 25 ordered pairs  $(\frac{1}{\gamma_o}, \frac{1}{\gamma_i})$ . Then we averaged over the simulations to arrive at a set of “average” solution curves  $N_0(t)$ ,  $J_1(t)$ ,  $N_1(t)$ ,  $J_2(t)$ ,  $N_2(t)$  and  $J_3(t)$ . From this set we calculated the above distributions and did the following projection in time.

Using the plots of the solutions we estimated at what point in time,  $t_s$ , it would be appropriate to approximate future solutions by a line, assuming that the trends set in thirty years could be relied upon for up to about ten years. For all but the calculation for  $(\frac{1}{\gamma_o}, \frac{1}{\gamma_i}) = (5, 20)$  we used  $t_s = 10,000$  days. The data from this point used in time until the end of the simulation,  $t_{30}$ , we found the least squares best-fit line to the set of “average” solutions for  $t \in [t_s, t_{30}]$ . With the best fit line we can now project future populations reliably for about ten years. We are also in a position to answer our question about future costs and their purchasing power as far as the incarceration times’ impact on the active dealer population.

To estimate the total cost of jailing dealers for a fixed amount of time,  $t_p$  in days, after  $t_{30}$  we must estimate the cost of jailing the  $i^{\text{th}}$  time offender population for  $t_p$  days, for  $i = 1, 2$  and  $3$ . Summing over  $i$  would give us the total jail cost for  $t_p$  days after  $t_{30}$ . To estimate the jail cost for the  $i^{\text{th}}$  time offender we must take the number of incarcerated dealers at time  $t \in [t_{30}, t_{30} + t_p]$ ,  $J_i(t) = m_i t + b_i$ , where  $m_i$  and  $b_i$  are the best fit line coefficients obtained earlier for the  $i^{\text{th}}$  time offender population and multiply it by the daily per inmate incarceration cost,  $c$ . Then  $cJ_i(t)$  is the cost for jailing the  $i^{\text{th}}$  time offender population for day  $t$ . Summing over  $t \in [t_{30}, t_{30} + t_p]$  we get the cost of jailing the  $i^{\text{th}}$  time offender population for  $t_p$  days. So if  $C_i(t_p, c)$  is the cost of jailing the  $i^{\text{th}}$  time offender population for  $t_p$  days, then

$$\begin{aligned}
C_i(t_p, c) &= \sum_{t=t_{30}}^{t_{30}+t_p} cJ_i(t) = c \sum_{t=t_{30}}^{t_{30}+t_p} m_i t + b_i \\
&= c \left( m_i \sum_{t=t_{30}}^{t_{30}+t_p} t + b \sum_{t=t_{30}}^{t_{30}+t_p} 1 \right) \\
&= cm_i \left( \frac{(t_{30}+t_p)(t_{30}+t_p+1)}{2} - \frac{(t_{30}-1)(t_{30})}{2} \right) + cb(t_{30}+t_p-t_{30}+1) \\
&= \frac{cm_i}{2} ((t_{30}+t_p)(t_{30}+t_p+1) - (t_{30}-1)(t_{30})) + cb(t_p+1)
\end{aligned}$$

So then if  $T(t_p, c)$  is the total cost of jailing dealers for  $t_p$  days after  $t_{30}$  then

$$\begin{aligned}
T(t_p, c) &= \sum_{i=1}^3 C_i(t_p, c) = \sum_{i=1}^3 \left( \frac{cm_i}{2} ((t_{30}+t_p)(t_{30}+t_p+1) - (t_{30}-1)(t_{30})) + cb(t_p+1) \right) \\
&= \frac{1}{2} c(m_1+m_2+m_3) ((t_{30}+t_p)(t_{30}+t_p+1) - (t_{30}-1)(t_{30})) + 3cb(t_p+1)
\end{aligned}$$

Now if we wish to know how much the active dealer population changes over the  $t_p$  days after  $t_{30}$  we can look at the percent change between the total active dealer population at time  $t_{30}$ ,

$$\text{old} = \sum_{j=0}^2 N_j(t_{30})$$

and the total active dealer population at time  $t_{30} + t_p$ ,

$$\text{new} = \sum_{j=0}^2 N_j(t_{30} + t_p).$$

Then the percent change is of course

$$\% \Delta = \frac{\text{new-old}}{\text{old}} \times 100\%.$$

## RESULTS and CONCLUSIONS

We performed the above calculations using an annual per inmate cost of \$21,000 (CDC web) and projections of  $t_p = 5$  years and 10 years for the set  $(\frac{1}{\gamma_o}, \frac{1}{\gamma_1}) \in [0.5, 5, 10, 15, 20] \times [0.5, 5, 10, 15, 20] = \Gamma$ . This means that  $c = 21,000/365.25$  (\$/day) and  $t_p = 5(365.25)$  and  $10(365.25)$ . The following pages contain the results.

You may notice at first that for  $(\frac{1}{\gamma_o}, \frac{1}{\gamma_1}) = (0.5, 0.5-20)$  we get  $T(t_p, c)$  much higher than any other group of incarceration times. One can immediately point to the amounts of money being spent in  $J_3$ . This is a result of a low first time incarceration that results in the inflow of a large number of fresh

dealers that must go through the system. With the long period of time in which a dealer can expect to stay in jail for his/her third time, inmates begin to accumulate and this accounts for the large amounts of money being spent on third time offenders,  $J_3$ . Now as the second incarceration time increases the amount of money spent on second time offenders also increases, and one notices that the most money is spent on  $J_2$  when  $\frac{1}{\gamma_1} = 20$  years. This results in  $(0.5, 20)$  having the highest  $T(t_p, c)$ . Notice that for all this money being spent (the highest throughout  $\Gamma$ !) the percent changes in this group do not compete with other points in  $\Gamma$ . Also notice that the projection for ten years just pronounces any differences in the data. We can conclude that there does not exist a direct relationship between money spent incarcerating dealers and a resultant drop in active dealer population.

Next notice that for the lowest amount of money, \$158.2 million, spent one gets one of the lowest positive percent change in active dealers. This remains true for the ten year projection. Although we do not get a drop in active dealer population we do get a smaller increase than if we had incarcerated under a more expensive policy. Again we can conclude that there does not exist a direct relationship between money spent incarcerating dealers and a resultant drop in active dealer population.

We then notice an interesting fact, there is a set  $\Omega$ ,

$$\Omega = \{(\frac{1}{\gamma_o}, \frac{1}{\gamma_1}) \in \Gamma : \frac{1}{\gamma_o} + \frac{1}{\gamma_1} = c, \text{ca constant.}\}$$

For our finite  $\Gamma$  we must get that  $\Omega$  is finite. Then it is easily verified that one will spend the least amount of money when  $\frac{1}{\gamma_o}$  is the largest value with  $(\frac{1}{\gamma_o}, \frac{1}{\gamma_1})$  still in  $\Omega$ . This could be helpful for policy makers who would like to claim that they put criminals away for no shorter than  $c$  years while maintaining a jail budget under some amount of money.

One may also notice that there are points in  $\Gamma$  that yield negative percent changes corresponding to drops in active dealers. The biggest of such drops occurs for  $(\frac{1}{\gamma_o}, \frac{1}{\gamma_1}) = (20, 15)$ ,  $\% \Delta = -8.13\%$  for the 5 year projection and  $\% \Delta = -15.966\%$  for the 10 year projection. Although this may seem great, you should then notice that incarcerating a first time offender for twenty years is really harsh and may be considered cruel and unusual punishment. This policy is not likely to be adopted by any municipality in Los Angeles county much less any part of the United States.

One of the most promising results may be the existence of such a point as  $\gamma^* = (10, 0.5)$ . This point exhibits some desirable qualities. It has a negative percent change, which means that it has a positive effect for society. Another

key quality is its frugality. For both the five and ten year projection the  $T(t_p, c)$  ranks 12<sup>th</sup> lowest in cost. The existence of such a point gives hope that there may be more such points, maybe within close proximity.

After some investigations with this model we have seen some dynamics that are not intuitive. This should lead one to believe that the problem of incarcerating dealers is not as clear cut as one might think at first. With improvements to our model and some more investigations we may find a solution to the problem of drug dealing in our nation.

### Improvements:

Our recommendations for future works on the stocastics models of Drugs Distribution is the consideration of the (or some of them) following:

- Criminals Rates : having under consideration that a drug dealer can be put in jail, by the commitment of another type of crime diferent from drug violation.
- Populations Interactions : divide the total population in non drugs users, drug users, and drugs dealers. Establish the relation between them, and study the dynamics.
- Jail Capacity : In reality the jails system support a finite number of criminals.
- Drug use and distribution depend on the space area and population density.
- In our model, add a chance that a criminal does not go back to dealing drugs after they are released the first time and after they are released the second time
- Perform the above simulations and calculations for postive integer less than 20 that we omitted to find other points such as  $\gamma^*$ .

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### Works Cited:

Torrance Police Department (anonymous).Telephone interview.11 July 1998.

Carson Narcotics Unit (anonymous).Telephone interview.11 July 1998.

"Los Angeles Times Communities" Welcome to Communities.1998.L.A.Times. 7 July 1998 <<http://www.latimes.com/HOME/COMMUN/>>.

"Maps".California Travel and Tourism.1998.California Division of Tourism. 7 July 1998 <<http://gocalif.ca.gov/>>.

Dan Lungren. "Three Strikes and Your Out....after Four Years" CA Dept. of Justice.1998.Office of the Attorney General. 21 July 1998 <<http://www.caag.state.ca.us/piu/3strikes/3strikes.html>>.

"Cost to a Society" NIDA Infobox.16.July 1998.National Institute of Health. 16 July 1998 <<http://www.nida.nih.gov/infobox/costs.html>>.

“The Drug Problem” National Drug Policy. 1998. Office of National Drug Policy. 21 July 1998 <<http://www.whitehousedrugpolicy.gov/drugfact/drugprob.html>>.

AskNCJRS. Information and help .askncjrs@ncjrs.org. 13 June 1998.

“About CDC”. CDC. 1998. About Prisoners. 2 August 1998  
<<http://www.cdc.state.ca.us/factsht.htm>>.